

B-7-X

Roll No.

Total No. of Questions : 31]

[Total No. of Printed Pages : 8

12thARM(SZ)JKUT2024

1107-X

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

SECTION-A

**(OBJECTIVE TYPE QUESTIONS/
MULTIPLE CHOICE QUESTIONS)**

1 each

1. In determinant

$$\begin{vmatrix} 3 & 6 \\ -2 & 5 \end{vmatrix}$$

cofactor of -2 is :

(A) 6

(B) 3

(C) 5

(D) -6

2. For a square matrix A. $A(\text{adj } A) = (\text{adj } A) A = \dots\dots\dots$

Turn Over

12thARM(SZ)JKUT2024-1107-X

B-7-X

3. Order of differential equation :

$$\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} = \frac{d^2y}{dx^2}$$

is :

- (A) 3
- (B) 2
- (C) 1
- (D) 4

4. Derivative of $e^{\log \sin x}$ is :

- (A) $\sin x$
- (B) $\cos x$
- (C) $\sec x$
- (D) $\tan x$

5. Number of points of discontinuity of the function :

$$f(x) = \frac{1}{x-5}, x \neq 5$$

is :

- (A) 1
- (B) 2
- (C) 3
- (D) 5

6. $\int x e^{x^2} dx$ is equal to $\frac{e^{x^2}}{x} + c$. (True/False)

7. The function $f(x) = 3x + 5$ is increasing for

8. Direction cosines of x-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. Magnitude of Vector $\hat{i} + 2\hat{j} + 3\hat{k}$ is :

(A) $\frac{1}{\sqrt{14}}$

(B) $\sqrt{14}$

(C) 1

(D) 10

10. Define objective function.

Turn Over

SECTION-B**(VERY SHORT ANSWER TYPE QUESTIONS) 2 each**

11. Check the injectivity and subjectivity of the function $f : \mathbb{C} \rightarrow \mathbb{C}$ given by $f(x) = x^2$.

12. Find the principal value of

$$\sin^{-1}\left(-\frac{1}{2}\right).$$

13. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

14. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. <https://www.jkboseonline.com>

15. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

16. Find :

$$\int \frac{(\log x)^2}{x} dx.$$

17. Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}.$$

12thARM(SZ)JKUT2024-1107-X

B-7-X

(5)

18. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events.

19. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

20. Show that the matrix :

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

21. Find the general solution of the differential equation :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

22. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

Turn Over

23. If

$$x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu (2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda \vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ .

26. Solve the linear programming problem graphically :

$$\text{Minimize } z = -3x + 4y$$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0, y \geq 0.$$

27. Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.

28. A bag contains 4 red and 4 black balls. another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION-D

(LONG ANSWER TYPE QUESTIONS)

6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify $A (\text{adj } A) = (\text{adj } A)A = |A| I$.

Turn Over

(8)

Or

Let :

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

30. Evaluate :

$$\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx.$$

Or

Evaluate :

$$\int \frac{(3x-1) dx}{(x-1)(x-2)(x-3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$

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12thARM(SZ)JKUT2024-1107-X

8-7-X

B-7-Y

Page No. _____

Total No. of Questions : 30

Total No. of Printed Pages : **8**

12th ARM(SZ)JKUT2024

1107-Y

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

SECTION-A

**(OBJECTIVE TYPE QUESTIONS/
MULTIPLE CHOICE QUESTIONS)**

1 each

1. In determinant

$$\begin{vmatrix} -1 & 3 \\ 6 & 10 \end{vmatrix}$$

cofactor of 10 is

(A) 1

(B) -1

(C) 3

(D) 6

2. If matrix B is inverse of matrix A, then $AB = BA = \dots\dots\dots$

Turn Over

12th ARM(SZ)JKUT2024-1107-Y

B-7-Y

3. Order of differential equation

$$x^2 \frac{dy}{dx} = -y^5$$

is

(A) 1

(B) 2

(C) 5

(D) 0

4. Derivative of $\cos (xe^x)$ is :

(A) $-e^x (x + 1) \sin (xe^x)$

(B) $e^x (x + 1) \sin (xe^x)$

(C) $-\sin (xe^x)$

(D) $\sin (xe^x)$

5. Number of points of discontinuity of the function $f(x) = x + 5$ is :

(A) 1

(B) 2

(C) 3

(D) 0

6. $\int \tan x \, dx$ is equal to $\log \sec x + c$. (True/False)

7. The function $f(x) = x^2 + 2x + 3$ is decreasing for

8. Direction cosines of y-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. Unit vector along the direction of the vector $\hat{i} + \hat{j} + \hat{k}$ is :

(A) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

(B) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$

(C) $\frac{\hat{i} + \hat{j} + \hat{k}}{3}$

(D) $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$

10. Define feasible region.

12thARM(SZ)JKUT2024—1107—Y

B-7-Y

SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$.

12. Find the principal value of

$$\tan^{-1}(-\sqrt{3}).$$

13. Prove that the function $f(x) = \log \cos x$ is decreasing for

$$\left(0, \frac{\pi}{2}\right).$$

14. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

15. Show that :

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b}).$$

16. Find :

$$\int \frac{1}{x + x \log x} dx.$$

17. Evaluate :

$$\int_0^{\pi/2} \cos 2x dx.$$

$$(1/5)$$

8. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events

9. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

10. Show that the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

1. Find the general solution of the differential equation .

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

2. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} - \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu (2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda \vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ .

26. Solve the linear programming problem graphically

$$\text{Minimize } z = -3x + 4y$$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12.$$

$$x \geq 0, y \geq 0.$$

27. Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.
28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION-D

(LONG ANSWER TYPE QUESTIONS)

6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify $A (\text{adj } A) = (\text{adj } A)A = |A| I$.

Turn Over

(8)

Or

Let

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

30. Evaluate :

$$\int_{-1}^1 5x^4 \sqrt{x^5 + 1} \, dx.$$

Or

Evaluate :

$$\int \frac{(3x-1) \, dx}{(x-1)(x-2)(x-3)}.$$

21. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$

12thARM(SZ)JKUT2024-1107-Y

B-7-Y

B-7-Z

Roll No.

Total No. of Questions : 31]

[Total No. of Printed Pages : 8

12thARM(SZ)JKUT2024**1107-Z****MATHEMATICS**

Time : 3 Hours]

[Maximum Marks : 80

SECTION-A**(OBJECTIVE TYPE QUESTIONS/
MULTIPLE CHOICE QUESTIONS)**

1 each

1. In determinant

$$\begin{vmatrix} -1 & 2 \\ -6 & 7 \end{vmatrix}$$

cofactor of 2 is :

(A) 6

(B) -6

(C) -1

(D) 7

2. If A and B are two matrices, then $(AB)' = \dots\dots\dots$ 12thARM(SZ)JKUT2024-1107-Z**B-7-Z**

Turn Over

3. Order of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

is :

(A) 1

(B) 2

(C) 4

(D) Not defined

4. Number of points of discontinuity of the function

$$f(x) = \frac{1}{x^2 - 5x + 6}, x \neq 2, 3$$

is :

(A) 1

(B) 2

(C) 3

(D) 0

5. $\int \sec x \, dx$ is equal to $\log (\sec x + \tan x) + c$. (True/False)

6. The function $f(x) = 2x^2 - 3x$ is decreasing for

12thARM(SZ)JKUT2024-1107-Z

B-7-Z

7. Derivative of $\cos(\sqrt{x})$ is :

(A) $\frac{-\sin(\sqrt{x})}{2\sqrt{x}}$

(B) $\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

(C) $2\sqrt{x} \cos(\sqrt{x})$

(D) $2\sqrt{x} \sin(\sqrt{x})$

8. Direction cosines of z -axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. $\hat{i} \cdot (\hat{j} \times \hat{k})$ is equal to :

(A) 1

(B) -1

(C) 0

(D) 2

10. Define Linear programming problem.

12thARM(SZ)JKUT2024-1107-Z

B-7-Z

Turn

SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$.

12. Find the principal value of

$$\cos^{-1} \left(\frac{\sqrt{3}}{2} \right).$$

13. Prove that the function $f(x) = \log \sin x$ is strictly increasing on

$$\left(0, \frac{\pi}{2} \right).$$

14. Evaluate the product : <https://www.jkboseonline.com>

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}).$$

15. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

16. Find $\int \sin x \sin (\cos x) dx$.

17. Evaluate :

$$\int_0^{\pi/4} \sin 2x dx.$$

18. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events.

19. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

20. Show that the matrix :

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

21. Find the general solution of the differential equation :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

22. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu (2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda \vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ .

12thARM(SZ)JKUT2024-1107-Z

B-7-Z

26. Solve the linear programming problem graphically

$$\text{Minimize } z = -3x + 4y$$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0, y \geq 0.$$

27. Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.
28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION-D

(LONG ANSWER TYPE QUESTIONS)

6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify $A (\text{adj } A) = (\text{adj } A)A = |A| I$.

Turn Over

Or

Let :

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1}.A^{-1}$.**30.** Evaluate :

$$\int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx.$$

Or

Evaluate :

$$\int \frac{(3x-1) dx}{(x-1)(x-2)(x-3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$

C-7-C

Roll No.

Total No. of Questions : 31]

[Total No. of Printed Pages : 8

12th ARA(SZ)JK/LEHUT2024
1507-C
MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

General Instructions :

1. This question paper contains four sections A, B, C and D. Each Section is compulsory.
2. Section A—Question 1 to 10 comprises of Very Very Short Answer Type Questions of 1 mark each.
3. Section B—Question 11 to 20 comprises of 10 Very Short Answer (VSA) Type Questions of 2 marks each.
4. Section C—Question 21 to 28 comprises of 8 Short Answer (SA) Type Questions of 4 marks each.
5. Section D—Question 29 to 31 comprises of Long Answer (LA) Type Questions of 6 marks each.

12th ARA(SZ)JK/LEHUT2024—1507-C

C-7-C

SECTION-A

(VERY VERY SHORT ANSWER TYPE QUESTIONS) 1 each

1. If

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

then x is equal to :

- (A) -2 (B) 2
(C) 3 (D) 4

2. If

$$A = \begin{vmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix}$$

and $A + A' = I$ then the value of α is :

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) π (D) $\frac{3\pi}{2}$

3. The number of arbitrary constants in the general solution of a differential equation of 4th order is :

- (A) 0 (B) 2
(C) 3 (D) 4

4. Second order derivative of $e^x \sin 5x$ is5. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to :

- (A) $\tan x + \cot x + C$ (B) $\tan x + \operatorname{cosec} x + C$
(C) $-\tan x + \cot x + C$ (D) $\tan x + \sec x + C$

6. The rate of change of the area of a circle with respect to its radius r when $r = 4$ is

7. $\int \frac{e^x(1+x)}{\cos^2(x.e^x)} dx$ is equal to :

(A) $-\cot(e^x.x) + C$

(B) $\tan(e^x.x) + C$

(C) $\tan(e^x) + C$

(D) $\cot(e^x) + C$

8. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$, then

$\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$

9. The vector equation of XZ plane is :

(A) $\vec{r} \cdot \hat{k} = 0$

(B) $\vec{r} \cdot \hat{j} = 0$

(C) $\vec{r} \cdot \hat{i} = 0$

(D) $\vec{r} \cdot \hat{d} = 0$

10. Define feasible region.

SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Determine the relation in the set Z of all integers defined as $R = \{(x, y) \mid x - y \text{ is an integer}\}$ is reflexive, symmetric and transitive.

12. Find the principal value of

$$\operatorname{cosec}^{-1}(2)$$

13. Find the interval in which the function $6 - 9x - x^2$ is strictly increasing or decreasing.

14. For given vectors :

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

and

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

15. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

16. Evaluate :

$$\int \frac{1 - \cos x}{1 + \cos x} dx$$

17. Using the properties of definite integrals, evaluate :

$$\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$$

18. Compute $P\left(\frac{A}{B}\right)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

19. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$. Find :

(i) $P(A \cap B)$

(ii) $P\left(\frac{A}{B}\right)$

20. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by :

$$a_{ij} = \frac{(i+2j)^2}{2}$$

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

21. Solve the differential equation :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

22. Evaluate using the properties of definite integrals .

$$\int_0^{\pi/4} \log(1 + \tan x) \, dx$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}$$

(6)

24. Find the shortest distance between the lines :

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}.$$

25. Show that the vectors

$$2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$$

and

$$3\hat{i} - 4\hat{j} - 4\hat{k}$$

form the vertices of a right-angled triangle.

26. Solve the following linear programming problem graphically :

Maximize : <https://www.jkbboseonline.com>

$$Z = 5x + 3y$$

subject to :

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0, y \geq 0$$

27. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by :

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \forall n \in \mathbb{N}$$

State whether the function is bijective, justify your answer.

28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION-D

(LONG ANSWER TYPE QUESTIONS)

6 each

29. Solve the following system of equation by matrix method :

$$2x + y + z = 1,$$

$$x - 2y - z = \frac{3}{2},$$

$$3y - 5z = 9$$

Or

Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of symmetric and skew-symmetric

matrix.

(8)

30. Evaluate the integral

$$\int \frac{2x-3}{(x^2-1)(2x+3)} dx$$

Or

Evaluate

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$$

31. If $y = (\tan^{-1} x)^2$, then show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

Or

Differentiate w.r.t. x :

$$x^x \cos x + \frac{x^2 + 1}{x^2 - 1}.$$