B–7–Z

Roll No. .....

Total No. of Questions : 31] [Total No. of Printed Pages : 8

12th ARF(SZ)JKUT2024-25
307–Z
MATHEMATICS

Time : 3 Hours] [Maximum Marks : 80

SECTION–A
(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

1. If

$$\begin{vmatrix} x & -2 \\ 6 & 4 \end{vmatrix} = 4$$

then x is equal to :

(A) –2

(B) 2

(C) 6

(D) 4

2. A square matrix A is singular if |A| = ...

3. Derivative of $e^{\log \sin x}$ w.r.t. x is :

(A) $\cos x$

(B) $-\cos x$

(C) $\sin x$

(D) $-\sin x$

4. Second derivative of x^{20} w.r.t. x is :

(A) $380\ x^{18}$

(B) $190\ x^{18}$

(C) $20\ x^{18}$

(D) $19\ x^{18}$

5. The function $f(x) = 5x - 3$ is continuous at $x = 0$. (True/False)

6. Maximum value of $\sin 2x + 5$ is :

(A) 5

(B) 4

(C) 6

(D) 7

7. $\int \sec x \, dx$ is equal to :

(A) $\log (\sec x + \tan x) + C$

(B) $\log (\sec x - \tan x) + C$

(C) $\log (\cot x + \text{cosec } x) + C$

(D) $\log (\cot x + \text{cosec } x) + C$

8. If two lines are intersecting, then shortest distance between them = ...

9. Projection of $i + \hat{j}$ on $\hat{i} - \hat{j}$ is zero. (True/False)

10. Define feasible region.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as $R = \{(a, b) \mid b = a + 1\}$ is reflexive, symmetric or transitive.

12. Prove that :

$$\sin^{-1} (2x \sqrt{1 - x^2}) = 2 \sin^{-1} x$$

$$-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

13. Find the rate of change of the area of a circle with respect to its radius 'r' when $r = 3$ cm.

14. Find $|\vec{a}|$ and $|\vec{b}|$, if

$$(\vec{a} + \vec{b}) . (\vec{a} - \vec{b}) = 8$$

and

$$|\vec{a}| = 8|\vec{b}|$$

15. If

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

and

$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

find the unit vector in the direction of $\vec{a} + \vec{b}$.

16. Find :

$$\int \frac{\cos x}{\sqrt{1 + \sin x}} dx$$

17. Evaluate :

$$\int_0^{\pi/4} \tan x \, dx$$

18. A coin is tossed three times. If E is the event 'head on third toss' and F is the event 'heads on first two tosses'. Find P(E | F).

19. Let E and F be two events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent ?

20. Find the product :

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

21. Find the relationship between a and b so that the function f defined by

$$f(x) = ax + 1 \text{ if } n \leq 3$$

$$= bx + 3 \text{ if } x > 3$$

is continuous at $x = 3$.

22. Evaluate :

$$\int_0^1 x(1-x)^n \, dx$$

23. Differentiate $(\log x)^x + x^{\log x}$ w.r.t. x.

24. Find the shortest distance between the lines :

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$$

and

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that :

$$\vec{a} + \vec{b} + \vec{c} = 0$$

find the value of

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

26. Solve the linear programming problem graphically :

Maximize $z = 3x + 2y$

Subject to :

$$x + 2y \leq 10,$$

$$3x + y \leq 15,$$

$$x, y \geq 0.$$

27. Let A and B be two sets. Show that :

$$f : A \times B \rightarrow B \times A$$

such that $f(a, b) = (b, a)$ is bijective function.

28. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?

SECTION–D
(LONG ANSWER TYPE QUESTIONS)

6 each

29. For a matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show $A^3 - 6A^2 + 5A + 11I = 0$.

Or

Solve the system of linear equations, using matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

30. Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$$

Or

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1 - x^2}} \, dx$$

31. Find the local maxima and local minima, if any, of the function :

$$f(x) = \sin x - \cos x, \; 0 < x < 2\pi.$$

Or

Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

B–7–Y

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307–Y
MATHEMATICS

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SECTION–A
(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

1. If

$$\begin{vmatrix} x & 6 \\ 3 & x \end{vmatrix} = 0$$

then x is equal to :

(A) $3\sqrt{2}$

(B) $-3\sqrt{2}$

(C) $\pm 3\sqrt{2}$

(D) 18

2. If B is the inverse of matrix A, then AB = ...

3. Derivative of $e^{\log x}$ w.r.t. x is :

(A) x

(B) $\log x$

(C) 1

(D) 0

4. Second derivative of $\tan^{-1} x$ w.r.t. x is :

(A) $\frac{1}{1+x^2}$

(B) $\frac{-2x}{(1+x^2)^2}$

(C) $\frac{2x}{1+x^2}$

(D) $\frac{2x}{(1+x^2)^2}$

5. Every polynomial function is continuous. (True/False)

6. Maximum value of $\cos x$ is :

(A) 1

(B) -1

(C) 2

(D) -2

7. $\int \cot x \, dx$ is equal to :

(A) $\log \cos x + C$

(B) $\log \sec x + C$

(C) $\log \sin x + C$

(D) $\log \operatorname{cosec} x + C$

8. Two lines are perpendicular if sum of products of their direction ratios = ...

9. Projection of $\hat{i}$ on $\hat{k}$ is 1 (True/False)

10. Define objective function.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Determine whether the following relation :

$$R = \{(x, y) \mid 3x - y = 0, x, y \in A\}$$

where $A = \{1, 2, 3, ..., 13, 14\}$ is reflexive, symmetric and transitive.

12. Prove that :

$$3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x);$$

$$x \in \left[\frac{1}{2}, 1\right]$$

13. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing ?

14. Find a vector in the direction of vector

$$5\hat{i} - \hat{j} + 2\hat{k}$$

which has magnitude 8 units.

15. Find the angle between the vectors

$$\hat{i} - 2\hat{j} + 3\hat{k}$$

and

$$3\hat{i} - 2\hat{j} + \hat{k}$$

16. Find :

$$\int \frac{\sin^2 x}{1 + \cos x} dx$$

17. Evaluate :

$$\int_2^3 \frac{dx}{x^2 - 1}$$

18. A coin is tossed three times. If E is the event 'head on third toss' and F is the event 'heads on first two tosses'. Find $P(E \mid F)$.

19. Let E and F be two events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent ?

20. Find the product :

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

21. Find the relationship between a and b so that the function f defined by

$$f(x) = ax + 1 \text{ if } n \leq 3$$

$$= bx + 3 \text{ if } x > 3$$

is continuous at $x = 3$.

22. Evaluate :

$$\int_0^1 x(1-x)^n \, dx$$

23. Differentiate $(\log x)^x + x^{\log x}$ w.r.t. x.

24. Find the shortest distance between the lines :

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$$

and

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that :

$$\vec{a} + \vec{b} + \vec{c} = 0$$

find the value of

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

26. Solve the linear programming problem graphically :

Maximize $z = 3x + 2y$

Subject to :

$$x + 2y \leq 10,$$

$$3x + y \leq 15,$$

$$x, y \geq 0.$$

27. Let A and B be two sets. Show that :

$$f : A \times B \rightarrow B \times A$$

such that $f(a, b) = (b, a)$ is bijective function.

28. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

29. For a matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show $A^3 - 6A^2 + 5A + 11I = 0$.

Or

Solve the system of linear equations, using matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

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30. Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$$

Or

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1 - x^2}} dx$$

31. Find the local maxima and local minima, if any, of the function :

$$f(x) = \sin x - \cos x, \; 0 < x < 2\pi.$$

Or

Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

B–7–X

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307–X
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SECTION–A
(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

1. If

$$\begin{vmatrix} x & 3 \\ 4 & -2 \end{vmatrix} = 0$$

then x is equal to :

(A) 6

(B) –6

(C) 4

(D) –4

2. For a square matrix A, A(adj A) = ...

3. Derivative of $e^{\log \tan x}$ w.r.t. x is :

(A) $\cot x$

(B) $\tan x$

(C) $\sec^2 x$

(D) $\operatorname{cosec}^2 x$

4. Second derivative of $\log (\log x)$ w.r.t. x is :

(A) $\dfrac{1}{x \log x}$ (B) $\dfrac{1}{\log x}$

(C) $\dfrac{1}{x}$ (D) $\dfrac{-(1 + \log x)}{(x \log x)^2}$

5. The function :

$$f(x) = \frac{|x|}{x}, \quad x \neq 0$$

$$= 0 , \quad x = 0$$

is discontinuous at $x = 0$. (True/False)

6. Maximum value of $\sin x$ is :

(A) 1

(B) -1

(C) 2

(D) -2

7. $\int \tan x \, dx$ is equal to :

(A) log cos x + C

(B) log sec x + C

(C) log sin x + C

(D) log cosec x + C

8. Two lines are parallel if their direction ratios are ..... .

9. Projection of $\hat{i}$ on $\hat{j}$ is zero. (True/False)

10. Define optimal solution.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Show that the relation R in the set {1, 2, 3} given by

$$R = \{(1, 2), (2, 1)\}$$

is symmetric but neither reflexive nor transitive.

12. Prove that :

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$$

$$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

13. The length 'x' of a rectangle is decreasing at the rate of 3 cm/minute and the width 'y' is increasing at the rate of 2 cm/minute. When $x = 10$ cm and $y = 6$ cm, find the rate of change of the perimeter.

14. Find the unit vector in the direction of the vector

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

15. Find $|\vec{x}|$, if for a unit vector $\vec{a}$,

$$(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 12$$

16. Find :

$$\int \frac{\sin x}{1 + \cos x} dx$$

17. Evaluate :

$$\int_2^3 \frac{x\, dx}{x^2 + 1}$$

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